Code M381 | Level 3 | 30 points |

Number theory and mathematical logic

Quick facts

No residential school

4 Tutor-marked assignments (TMAs)

Examination

Course content

The course will give you an insight into two branches of very pure mathematics that have both historical and philosophical significance. By the end of it you should feel confident to tackle number-theoretic problems and have an appreciation of the nature and limitations of mathematics.

The course consists of two independent sections that are studied concurrently. Each has its own course texts and written work.

Number theory This section is concerned with the integers, and in particular with the solution of classical problems that require integer solutions. It begins by considering some elementary properties of the integers, such as divisibility and greatest common divisors. This leads to a method of solving the linear Diophantine equation ax + by = c, that is, finding solutions to the equation that are integers.

Every integer greater than 1 is shown to be a unique product of primes, and some results are obtained concerning the distribution of primes among the integers. In the theory of congruences, methods are developed for solving linear congruences such as ax ≡ b {mod n) and the classical theorems of Fermat and Wilson are obtained. We then consider multiplicative functions: functions f satisfying f(m) x f(n) = f(mn) for relatively prime integers m and n, and in particular Euler's φ-function, which counts the number of integers in the set { 0, 1, …, (n−1)} that are relatively prime to n. Returning to congruences we consider the solution of quadratic congruences, which leads to Gauss's law of quadratic reciprocity. Finally, the story of continued fractions is developed and applied as a method of solving further examples of Diophantine equations.

Mathematical logic This section looks at theoretical issues concerning algorithms and what they can compute, and at a theory of mathematical reasoning. One of the major questions we address is whether there is an algorithm for deciding which statements of number theory are true. On the way to answering this, we discuss two different abstract notions of computable functions: those arising from unlimited register machines and those from the theory of recursive functions. First we show that these notions of computability give rise to the same class of computable functions, give evidence towards Church's thesis on computability and lead to results about limitations on what can be computed. Then we look at the formalisation of a mathematical language for number theory, and at a formal proof system for it. Finally, the material so far is combined to give proofs of Gödel's incompleteness theorem, a result of great philosophical importance for the limits of mathematical endeavour.

Entry

This is a Level 3 course. Level 3 courses build on study skills and subject knowledge acquired from studies at Levels 1 and 2. They are intended only for students who have recent experience of higher education in a related subject, preferably with The Open University. The course assumes no specific mathematical knowledge beyond A-level or Scottish Highers in pure mathematics, but many of the concepts require considerable mathematical sophistication, such as facility in reading mathematical arguments and some experience of producing them, as developed in either of the Open University Level 2 mathematics courses M208 *Pure mathematics* or MST209 *Mathematical methods and models.* Students are more likely to complete this course successfully if they have acquired their prerequisite knowledge through passing these courses. If you have any doubt about the suitability of the course, please contact our Student Registration & Enquiry Service.

Preparatory work

There is no specific preparatory work required for this course. A flavour of what is involved in number theory can be obtained from books such as *Elementary Number Theory* by D.Burton, or *Elementary Number Theory* by Gareth Jones and Mary Jones*.* A flavour of the first part of the mathematical logic can be obtained from the book *Computability* by N. Cutland.

Qualifications

This is a specified course in our:

- B14 BSc (Hons) Computing and Mathematical Sciences
- B31 BSc (Hons) Mathematics

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- B36 BA (Hons) or BSc (Hons) Mathematics and Statistics
- B46 BSc (Hons) Mathematics and its Learning

It can also count towards most of our other degrees at bachelors level, where it is equally appropriate to a BA or BSc. We advise you to refer to the relevant award descriptions for information on the circumstances in which the course can count towards these qualifications because from time to time the structure and requirements of a qualification may change.

If you have a disability or additional requirements

Written transcripts are available for the audio-visual material. This course may be substantially challenging if you have impaired sight, but it is not impossible. If you are a new student, make sure that you have our booklet *Meeting your needs*. You can obtain a copy by contacting our Student Registration & Enquiry Service. We provide a range of support services for individual needs but some of these may take several months to arrange. Please contact us for advice if you have concerns about taking this course, or about the support that could be provided. The website www.open.ac.uk/disability has the latest information about availability.

Course materials

We use a mixture of media to help you learn. Our courses may include any of the following media that you will use from home (or wherever you choose to study): specially written texts, set books, online resources, audio CDs, audio and video cassettes, DVDs, CD-ROMs, computer software, a home experiment kit. For further information on set books go to http://www3.open.ac.uk/about/setbooks/index.shtm.

Teaching and assessment

Support from your tutor

You will have a tutor who will help you with the course material and mark and comment on your written work, and whom you can ask for advice and guidance. We may also be able to offer group tutorials or day schools that you are encouraged, but not obliged, to attend. Where your tutorials are held will depend on the distribution of students taking the course. Contact our Student Registration & Enquiry Service if you want to know more about study with the Open University before you register.

Assessment

There are four tutor-marked assignments (TMAs), submitted on paper, and an examination. Assessment is an essential part of the teaching, so you are expected to complete it all. But if you unavoidably miss or do badly in an assignment some courses allow you a 'substitution score', calculated as a weighted average of all your scores for the course. In M381 this rule can apply to one assignment only. You will be given more detailed information when you begin the course.

Professional recognition

This course may help you to gain recognition from a professional body. Ask our Student Registration & Enquiry Service for *Recognition* leaflet 3.6 *Institute of Mathematics and its Applications* or see the course description for **M381** at www.open.ac.uk/courses.

More information

For full details, including course fees, start dates, services for disabled students, any computing requirements and information on how to become a student, visit the Course and Qualifications website www.open.ac.uk/courses.

We make every effort to ensure that this information is accurate but it could change if regulations or policies change or because of financial or other constraints.

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